* **Capacitors** :→ It is a system of two conductors seperated by an insulator.

Both conductor's having charge Q_1 and Q_2 and potential V_1 and V_2 :

Cofiguration of Capacitor → { In which $Q_1 = -Q_2 = Q$. Potential Diffrence $V = V_1 - V_2$

* Single conductor can be used as a capacitor by assuming the other at infinite.

→ The electric field in the region ~~b/w~~ between the conductor's is proportional to the charge Q.

→ The potential develop between the conductor directly proportional to charge also.

$$V \propto Q$$

$$V = \frac{1}{C} Q$$

$$\boxed{Q = CV}$$

where C = Capacitance of capaciter.

→ Capacitance depend on geometrical configuration of system.

$$\boxed{C = \frac{Q}{V}}$$

S.I Unit of Capacitance = C/Volt
= Farad(F)

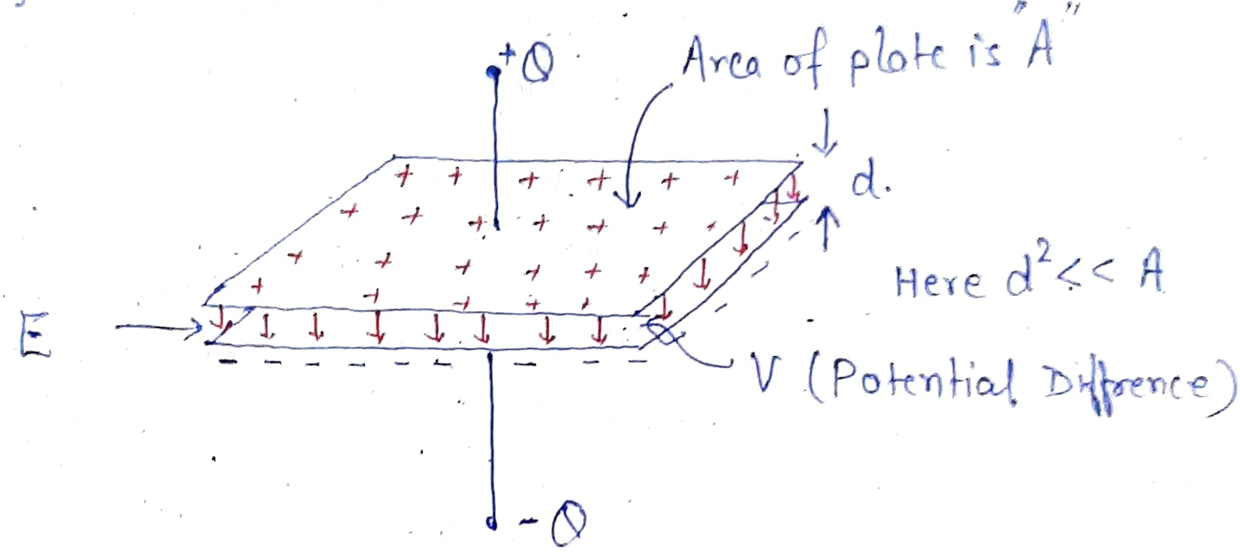
Capacitor of fixed capacitance = or

Capacitor of Variable capacitance = or

Types of Capacitor :→

1. Parallel Plate Capacitor.
2. Cylindrical Capacitor.
3. Spherical Capacitor.

1.* Parallel Plate Capacitor :→ It consist two large plane parallel conducting plates seprated by a small distance.



Consider Charge of plate is : $(Q, -Q)$

Seprating distance = d.

Area of plate = A

Surface Charge density = $(\sigma, -\sigma)$

then Field between plate is

$$E = E^+ - E^-$$

{ We know from application of Gauss law
$E = \sigma / 2\epsilon_0$ }

$$E = \frac{\sigma}{2\epsilon_0} - \left(-\frac{\sigma}{2\epsilon_0}\right)$$

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0} \quad \text{—— (1)}$$

Now we know $E = \frac{V}{d}$

and $\sigma = \frac{Q}{A}$

from eqn (1)

$$E = \frac{\sigma}{\epsilon_0}$$

$$\Rightarrow \frac{V}{d} = \frac{Q}{A\epsilon_0}$$

$$\Rightarrow \frac{A\epsilon_0}{d} = \frac{Q}{V}$$

$$\Rightarrow \frac{Q}{V} = \frac{A\epsilon_0}{d}$$

$$\boxed{C = \frac{A\epsilon_0}{d}}$$

→ Here we see that $\uparrow C \propto A \uparrow$

$$\downarrow C \propto \frac{1}{d\uparrow}$$

* Capacitance of parallel plate capacitor directly proportional to Area of plate and inversly proportional to distance between plate.

* Effects of Dielectric's on plarallel plate Capacitor:

Here we know $C = \frac{Q}{V}$

i.e For a large Capacitance V is small for a given Charge. Q. This means a capacitor with large Capacitance can hold large amount of Charge Q at a relatively small V.

→ But due to heigh potential diffrence, implies strong electric field;

→ A strong Electric field can ionise the surrounding air and accelerate the charge so produce oppositively charge plate.

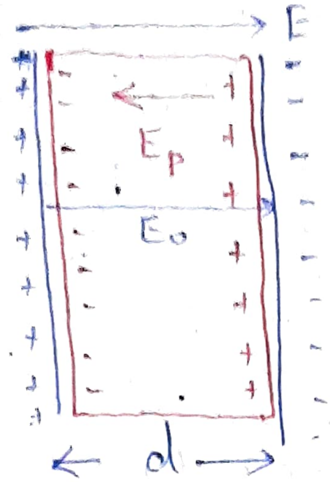
→ The Charge of the Capacitor leak away due to the reduction in insulating power of intervening medium.

→ The maximum electric field that a dielectric medium can withstand without breakdown is called dielectric Strength.

→ For a Capacitor to store large amount of Charge without leaking its Capacitance should be heigh enough so that potential diffrence and hence the Electric field do not exceed breakdown limit.

→ We put a dielectric slab ~~b~~ between the plate having dielectric Constant (K.) .

Now



According to above explanation net Field :-

$$\Rightarrow E = E_0 - E_p$$

$$\Rightarrow E = \frac{\sigma}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0}$$

$$\Rightarrow \cancel{E = \frac{\sigma}{\epsilon_0}\left(1 - \frac{\sigma_p}{\sigma\epsilon_0}\right)}$$

$$\Rightarrow E = \frac{\sigma}{\epsilon_0}\left(1 - \frac{\sigma_p}{\sigma}\right) \quad \text{——— (I)}$$

We Know $K = \dfrac{E_0}{E_0 - E_p}$

$$K = \frac{\sigma/\epsilon_0}{\sigma/\epsilon_0 - \frac{\sigma_p}{\epsilon_0}}$$

$$K = \frac{\sigma/\cancel{\epsilon_0}}{\sigma/\cancel{\epsilon_0}\left(1 - \frac{\sigma_p}{\sigma}\right)}$$

$$\frac{1}{K} = \left(1 - \frac{\sigma_p}{\sigma}\right) \quad \text{——— (II)}$$

from Eq^n (I) & (II)

$$\Rightarrow E = \frac{\sigma}{\epsilon_0} \cdot \frac{1}{K}$$

$$\Rightarrow \frac{V}{d} = \frac{Q}{A\epsilon_0 K}$$

$$\Rightarrow \frac{K A\epsilon_0}{d} = \frac{Q}{V}$$

$$\Rightarrow K C_0 = C$$

$$\Rightarrow \boxed{C = K C_0} \qquad K > 1$$

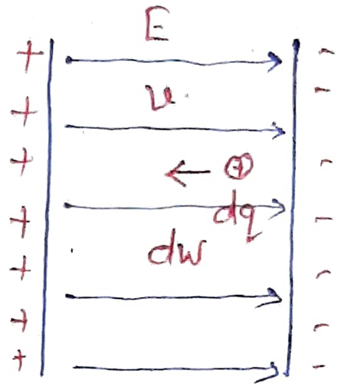
$$\Rightarrow \boxed{K = \frac{C}{C_0}}$$

* Here we see that the capacitance of Capcitor increased by "K" times.

Energy Stored in A Capacitor

In transfering charge from Conductor 2 to Conductor 1 work will done externaly.

To Calculate total work done we first Calculate the work done in a small step transfer of a very small amount of charge as :-



We know $V = \frac{dw}{dq}$

$\Rightarrow dw = v\,dq$

Now Integrating we get

$$\Rightarrow \int_0^U dw = \int_0^Q v\,dq$$

$$\Rightarrow [W]_0^U = \int_0^Q \frac{q}{C}\,dq \qquad \left\{ C = \frac{q}{V},\ V = q/c \right\}$$

$$\Rightarrow U - 0 = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\Rightarrow \boxed{U = \frac{Q^2}{2C}}$$

$$\because C = \frac{Q}{V} \Rightarrow Q = CV$$

$$\Rightarrow U = \frac{(CV)^2}{2C}$$

$$\boxed{U = \frac{1}{2}CV^2}$$

For Parallel. Plate Capacitor:-

$$C = \frac{A\epsilon_0}{d} \quad \& \quad V = E \cdot d.$$

then

$$\Rightarrow \quad U = \frac{1}{2} C V^2$$

$$\Rightarrow \quad U = \frac{1}{2} \frac{A\epsilon_0}{d} (Ed)^2$$

$$\Rightarrow \quad U = \frac{1}{2} (Ad)\, \epsilon_0 E^2$$

$$\Rightarrow \quad \frac{U}{Ad} = \frac{1}{2} \epsilon_0 E^2$$

$$\Rightarrow \quad \frac{U}{\text{Volume}} = \frac{1}{2} \epsilon_0 E^2$$

$$\Rightarrow \quad \left\{ u \text{ (Energy Density)} = \frac{U}{\text{Volume}} \right\}$$

$$\Rightarrow \quad \boxed{u = \frac{1}{2} \epsilon_0 E^2}$$

* **Electric Displacement**:→ It is define as resulting field linear dependent on Electric field and Polarization "P". It is denoted by "D."

$$\boxed{D = \epsilon_0 E + P}$$

This show the main important role when we put dielectric slab between capacitor plate.

We Know $\vec{E} = \vec{E}_0 - \vec{E}_p$

$$\Rightarrow \quad \vec{E} = \frac{\sigma}{\epsilon_0}\hat{n} - \frac{\sigma_p}{\epsilon_0}.\hat{n}$$

{where $\vec{E} = E.\hat{n}$, $\sigma_p \hat{n} = P.\hat{n}$ }

$$\Rightarrow \quad \vec{E} = \frac{\sigma}{\epsilon_0}.\hat{n} - \frac{P}{\epsilon_0}.\hat{n}$$

$$\Rightarrow \quad E.\hat{n} = \left(\frac{\sigma - P}{\epsilon_0}\right).\hat{n}$$

$$\Rightarrow \quad \epsilon_0 E.\hat{n} = (\sigma - P).\hat{n}$$

$$\Rightarrow \quad P.\hat{n} + \epsilon_0 E.\hat{n} = \sigma.\hat{n}$$

$$\Rightarrow \quad \underbrace{(P + \epsilon_0 E)}_{\text{This is known as Electric Displacement } (D)}.\hat{n} = \sigma.\hat{n}$$

$$\Rightarrow \quad D.\hat{n} = \sigma.\hat{n}$$

i.e $\boxed{\vec{D} = \sigma.\hat{n}}$

Where $\boxed{D = P + \epsilon_0 E}$ —— (1)

The significance of D is this, In vacuum E is related to the free charge density σ. when a diectric medium is present.

Now $\dfrac{D}{E} = \dfrac{\sigma}{E_0 - E_p}$

$$\frac{D}{E} = \frac{\sigma}{\frac{\sigma}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0}}$$

$$\frac{D}{E} = \frac{\sigma \epsilon_0}{\sigma - \sigma_p} = \left(\frac{\sigma}{\sigma - \sigma_p}\right) \cdot \epsilon_0$$

$$\frac{D}{E} = K\epsilon_0$$

Now $\boxed{D = K\epsilon_0 E}$ — (II)

then from eqn (I) & (II) we get

$$K\epsilon_0 E = \epsilon_0 E + P$$

$$K\epsilon_0 E - \epsilon_0 E = P$$

$$(K-1)\epsilon_0 E = P$$

$$\boxed{P = (K-1)\epsilon_0 E} \quad \text{— (III)}$$

But we know $P = \chi_e E$ — (IV)

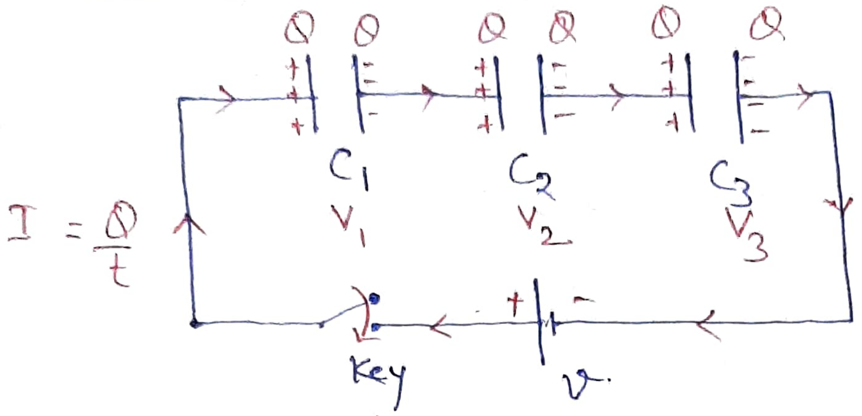
from eqn (III) & (IV) we get

$$\chi_e E = (K-1)\epsilon_0 E$$

Electric Susceptibility $\rightarrow$ $\boxed{\chi_e = (K-1)\epsilon_0}$

* Combination of Capacitor :→ Two or more than two capacitor grouped as diffrent way known as Combination of Capacitor :-

① Capacitors in Series :-



→ (-ve) terminal of 1st Connect with (+ve) terminal of second.

→ Charge flow through each Capaciter is same.

→ Potential devide among Capaciter.

Now $V = V_1 + V_2 + V_3$ —— ①

$\left\{ \text{but} \quad V = Q/C \right\}$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

for two Capaciter

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{C_2 + C_1}{C_1 \cdot C_2}$$

$$\boxed{C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}}$$

n- Capacitor

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_n}}$$

If all Capacity ~~of~~ will same = C

$$\frac{1}{C_{eq}} = \frac{n}{C} \Rightarrow \boxed{C_{eq} = \frac{C}{n}}$$

* Equivalent Capacity of these Capaciter is less than individual Capaity of each in series.

Voltage Division in each Capaciter is :-

$$V = V_1 + V_2$$

$$\left\{ \text{If } Q_1 = Q_2 \quad C_1 V_1 = C_2 V_2 \quad V_2 = \frac{C_1}{C_2} V_1 \right\}$$

$$V = V_1 + \frac{C_1}{C_2} V_1$$

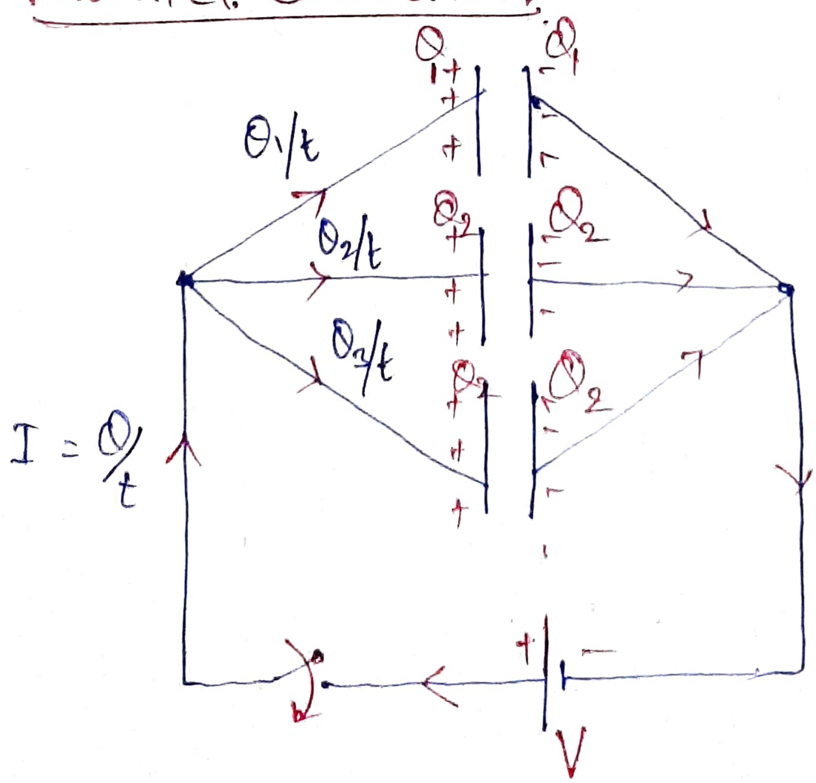
$$V = V_1 \left(\frac{C_2 + C_1}{C_2} \right)$$

$$\boxed{V_1 = \frac{V C_2}{C_1 + C_2}}$$

$$V_2 = \frac{C_1}{C_2} \cdot \frac{V C_2}{C_1 + C_2}$$

$$\boxed{V_2 = \frac{V C_1}{C_1 + C_2}}$$

(2) Parallel Combination :-



→ All (+ve) terminal of each Capacitor connect together and all (−ve) terminal of each Capacitor connect.

→ Charge will distributed among all capacitance.

→ Potential will same in all capacitance.

Now $Q = Q_1 + Q_2 + Q_3$ ——— (1)

$\{\because Q = CV\}$

$$C_{eq}V = C_1V + C_2V + C_3V$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

For n- Capacitor $\boxed{C_{eq} = C_1 + C_2 + C_3 - - + C_n}$

If all having same capacity

$$C_{eq} = C + C + C - - - \text{ n times}$$

$$\boxed{C_{eq} = nC.}$$

* Equivalent Capacity of these Capacitor is ~~less~~ greater than individual.

* **Charge Division Rule:-**

$$Q = Q_1 + Q_2$$

$\left\{ \text{If } V_1 = V_2 \quad \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \quad Q_2 = \frac{C_2}{C_1} Q_1 \right\}$

$$\Rightarrow Q = Q_1 + \frac{C_2}{C_1} Q_1$$

$$\Rightarrow Q = Q_1\left(1 + \frac{C_2}{C_1}\right)$$

$$\Rightarrow Q = Q_1 \frac{(C_1 + C_2)}{C_1}$$

$$\Rightarrow \boxed{Q_1 = \frac{QC_1}{C_1 + C_2}}$$

$$Q_2 = \frac{C_2}{C_1} \cdot Q_1$$

$$Q_2 = \frac{C_2}{\cancel{C_1}} \times \frac{Q\cancel{C_1}}{C_1 + C_2}$$

$$\boxed{Q_2 = \frac{QC_2}{C_1 + C_2}}$$